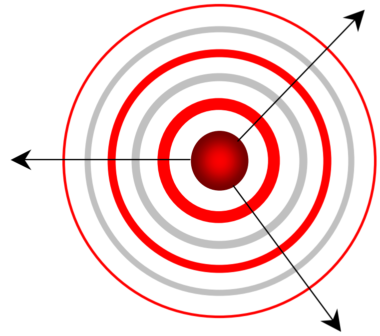
5. PHYSICS OF SOUND



Definition: **Sound** - a disturbance which propagates through an elastic material at a speed which is characteristic of that material. Sound is usually caused by radiation from a solid vibrating surface or fluid disturbances.

5.1 ACOUSTIC VARIABLES

As sound propagates through air (or any elastic medium), it causes measurable fluctuations in pressure, velocity, temperature and density. We can describe the physical state in terms of mean (steady state) values and small fluctuations about that mean. For our purposes in acoustics and noise control, all we care about is the fluctuating portion.

Physical Quantity	State Variable	Units
Pressure:	$P_{total} = P + p(r,t)$	(Pascals)
Velocity:	$U_{total} = U + u(r,t)$	(meters/second)
Temperature:	$T_{total} = T + \tau(r,t)$	(° Celsius)
Density:	$\rho_{total} = \rho + \sigma(r,t)$	(kg/m^3)

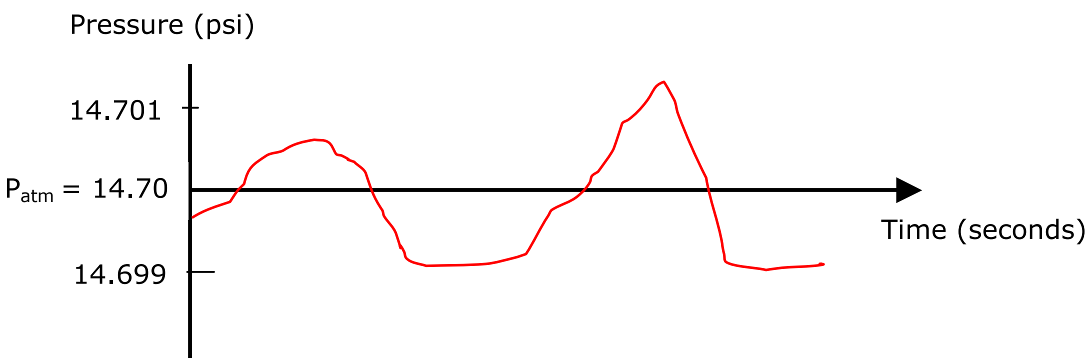


Figure 1. Variation of sound pressure about the mean value (1 atm)

P = 1 standard atmosphere = 14.7 psi = 1.013 bars = 1.013 x 10^5 Pa

Useful Conversion Factors:

1Pa = 1 N/m^2 = 10µ Bar
1 Psi = 6894 Pa

Typical sound pressure magnitudes range from just barely audible, 20 µPa or 3 x 10^{-9} psi (0 dB, the threshold of hearing) to the threshold of pain at approximately 60 Pa or .009 psi (130 dB). A typical conversation level is 0.1 Pa (74 dB).

It is easiest to measure sound pressure in air with a microphone. It is possible, but more difficult to measure acoustic velocity. That's why we mostly talk about sound pressure, because it's easiest to measure.

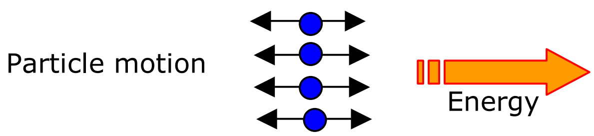
The ratio of pressure to particle velocity is a useful quantity and is called **Impedance**: – The specific acoustic impedance is the complex ratio (since both p and u are complex quantities) of the effective sound pressure at a point of an acoustic medium or mechanical device, to the effective particle velocity at that point. The unit is the mks rayl (Newton/cubic meter)

$$Z = \frac{p}{u}$$

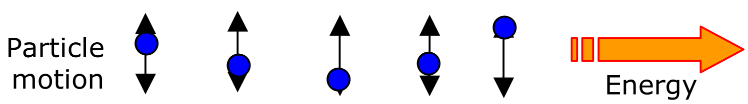
5.2 SOUND WAVES

The sound disturbance travels in space. There is energy transport (the disturbance propagates), but there is no net transfer of mass (no convection). Each particle in the fluid moves back and forth about one position. In general, sound waves in any medium can be a mixture of longitudinal and shear waves, depending primarily on the boundary conditions.
See http://www.kettering.edu/~drussell/Demos/waves/wavemotion.html for animations of longitudinal and shear waves.

Longitudinal Wave – Simplest type of wave is compressional (or longitudinal wave) where the particle oscillation is in the same direction as the energy transport. The disturbance propagates in the direction of the particle motion. This is the predominant mechanism in fluids and gases because shear stresses are negligible.



Shear Wave – The particle motion direction is orthogonal (perpendicular) to direction in which the disturbance (and the energy) propagates. In solids, you can have transverse shear and torsional waves. Bending waves (in a beam or plate), and water waves are a mixture of shear and longitudinal waves.



5.3 SPEED OF SOUND

For a longitudinal wave in an unbounded medium, sound travels at a speed of *c*:

$$\boxed{c = \sqrt{\frac{E}{\rho}}}$$

E = Young's modulus for a solid material, or the bulk modulus for a fluid
ρ= density of the material

$$\text{Bulk Modulus} = -\frac{V}{\partial V / \partial P} \qquad V = \text{Volume}$$

In normal gases, at audible frequencies, the pressure fluctuations occur under essentially adiabatic conditions (no heat is transferred between adjacent gas particles). Speed of sound then becomes:

$$c = \sqrt{\frac{\gamma P}{\rho}} \qquad \text{where}: \gamma = \frac{C_p}{C_v} = 1.4 \text{ for air} \quad \text{and } P = \rho RT \text{ (Ideal Gas Law)}$$

For gases, the speed of sound is solely a function of temperature and, to a smaller extent humidity since that changes the gas mixture and its density.*

$$c = \sqrt{\gamma RT} = 20.05\sqrt{T}\,(^\circ K) \quad \text{(meters/second)} \quad ^\circ\text{K} = ^\circ\text{C} + 273.15$$

$$= 49.03\sqrt{T}\,(^\circ R) \quad \text{(feet/second)} \quad ^\circ\text{R} = ^\circ\text{F} + 459.7$$

example: @ 20°C c = 343 m / sec or 1126 ft / sec

**Except when the acoustic pressure exceeds ~ 10 Pa, in which case the sound velocity varies with pressure. This is the realm of non-linear acoustics, which is beyond the scope of this course. This typically happens in sonic booms. (How many dB is 10 Pa?)*

Depending on what the propagation medium is, the sound speed can change with frequency.

Non-Dispersive Medium – Sound speed is independent of frequency, therefore the speed of energy transport and sound propagation are the same. Air is a non-dispersive medium.

Dispersive Medium – Sound speed is a function of frequency. The spatial and temporal distribution of a propagating disturbance will continually change. Each frequency component propagates at each its own phase speed, while the energy of the disturbance propagates at the group velocity: C_g. Water is an example of a dispersive medium.

5.4 WAVE EQUATION

Acoustic phenomena are generally associated with small fluctuations (linear acoustics), which are described mathematically by the linearized wave equation: $\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$.

5.5 PLANE WAVE PROPAGATION

For plane wave propagation (pressure varies in only one dimension, x), the linearized wave equation reduces to:

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

An example of a plane wave would be a speaker at the end of a long tube. If you poke around with a microphone you will find that the sound pressure is constant everywhere in the tube. If scan along a cross section of the pipe you will observe that all the particles along that cross section are moving in phase. In other words, the radiating wavefront is a plane. Other examples of plane wave propagation are if you have a large flat surface which is uniformly vibrating. In the immediate proximity of the surface, the sound pressure is constant. See http://www.acs.psu.edu/users/sparrow/movies/animations5.html for an animation of plane waves.

The general plane wave solution for pressure p is made up of waves traveling in both the positive and negative directions:

$$p(x,t) = f_1(t - \frac{x}{c}) + f_2(t + \frac{x}{c})$$

This physically corresponds to a pressure wave traveling in +x direction and a pressure wave traveling in –x direction.

The acoustic particle velocity is related to pressure by: $u(x,t) = \frac{1}{\rho c} p(x,t)$

The impedance is: $$Z = \frac{p(x,t)}{u(x,t)} = \rho c$$

(ρc sometimes called the characteristic impedance of the medium)

Since any function can be represented by Fourier analysis as a sum of harmonic pieces, a basic solution "building block" is a sinusoidal wave which propagates in the +x direction: $p(x,t) = A \sin[kx - \omega t + \beta]$

where: ω = frequency (radians/sec) β=phase angle (radians) $k = \omega/c$ = wavenumber

This is a wave, whose amplitude, A does not vary. The wave repeats each time the argument in the square brackets changes by 2πn radians. Such a change can be caused by variation in time t, distance x, or both. If we can hold time still and look at what happens in x, the spatial pattern repeats each time the distance has a value:

$$x_n = 2\pi n/k \qquad n = 1,2,3\ldots$$

If we could freeze time and look at pressure distribution in space (see Figure 2):

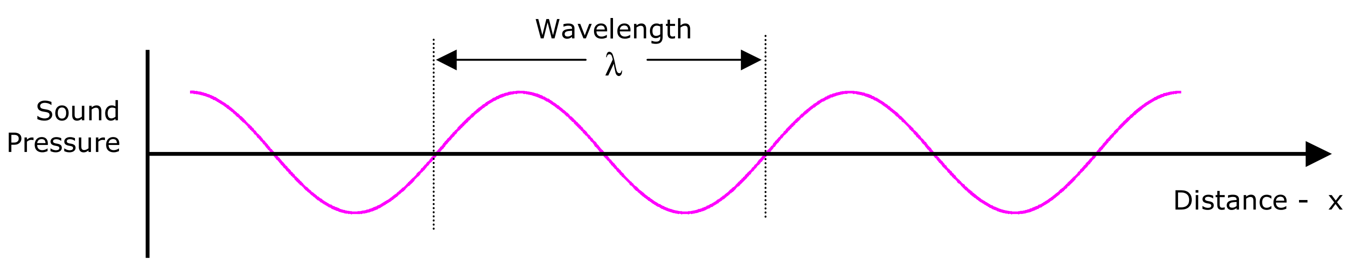


Figure 2. Spatial pressure distribution in a plane wave at one instant of time

We define the wavelength λ as the distance between repeating features of the wave:

$$\lambda = \omega/c = 2\pi/k$$

That takes care of the x part of the equation, now what about the time dependency? If we put a microphone at a fixed location in space (hold x constant in the equation) and view an oscilloscope trace of what the microphone hears, i.e., the time history, we see (Figure 3):

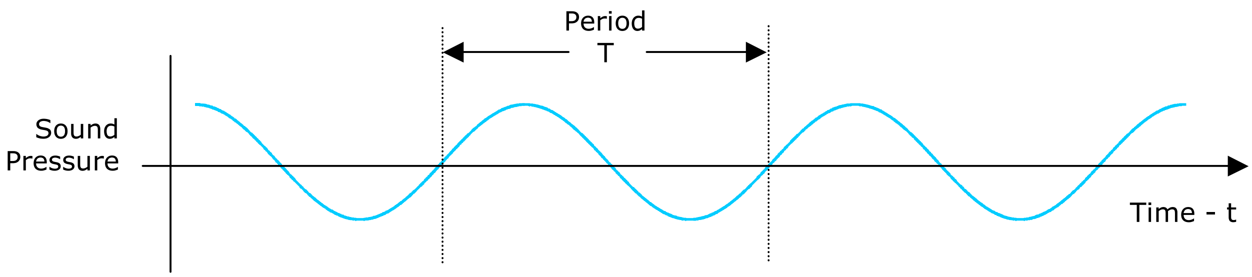


Figure 3. Time variation of pressure in a plane wave at one location in space

The time for wave to repeat (its period) $T = 2\pi/\omega = 1/f$

Another useful equation which relates frequency to wavelength is: $c = f\lambda$

5.6 PLANE WAVE INTENSITY

Acoustic pressure and particle velocity are in phase for a plane wave, and therefore, power is transmitted. ***Intensity*** is the power transmitted per unit surface area coincident with a wavefront at a fixed position in space. A general definition, valid for any geometry:

$$I = \lim_{T\to\infty} \frac{1}{T}\int_0^T pu\,dt \quad = \overline{pu}$$

(Note: the — above pu is shorthand notation for the long time average)

Since p and u are in phase and $p/u = \rho c$ for a plane wave, we can write: $I = \dfrac{\langle p^2 \rangle}{\rho c} = \rho c \langle u^2 \rangle$

(Note: the $\langle\,\rangle$ brackets are shorthand notation for the mean value)

The mean (of the) square acoustic pressure is: $\langle p^2 \rangle = \lim_{T\to\infty} \frac{1}{T}\int_0^T p^2 dt$

The r.m.s. pressure (root mean square) = $p_{rms} = \sqrt{\langle p^2 \rangle}$

A sound level meter usually measures p_{rms}.

(Note: For a sine wave A*sin*ωt, the rms level is .707A. What is the rms level of a square wave?) From now on, we will omit the *rms* subscript for brevity, so when we write p we really mean p_{rms}

Summary of Plane Waves:
- Plane waves radiate in one direction
- Pressure amplitude is constant over distance
- The sound intensity is proportional to p_{rms}^{2}
- Pressure and velocity are in phase (i.e. the impedance is real and = ρc)

5.7 SPHERICAL WAVE PROPAGATION

If we have a point sound source (the physical dimension of the source is much smaller than a wavelength), the sound pressure will be constant anywhere on a sphere surrounding the source. We also know from common sense that like light from a bulb, the sound pressure should diminish as we travel away from the source.
See http://www.acs.psu.edu/users/sparrow/movies/animations5.html for an animation of spherical waves

The wave equation in spherical coordinates for a uniformly radiating point source is:

$$\frac{\partial^2 (rp)}{\partial t^2} = c^2 \frac{\partial^2 (rp)}{\partial r^2}$$

The general solution is an outgoing and incoming wave with a radial distance dependence:

$$p(r,t) = \frac{1}{r} f_1\left(t - \frac{r}{c}\right) + \frac{1}{r} f_2\left(t + \frac{r}{c}\right)$$

(outgoing) (incoming)

The outgoing wave is similar to the plane wave case, but the magnitude now depends on the distance from the source: $p(r,t) = \frac{A}{r} \sin[\omega t - kr + \beta]$

The acoustic particle velocity is: $\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial r}$

Considering only the outgoing wave, the velocity is:

$$u(r,t) = \frac{1}{\rho r^2}\int_0^t dt + \frac{p(r,t)}{\rho c}$$

For large values of r (large in terms of a wavelength), the first term becomes negligible and the impedance converges to ρc. The curvature of the wave front becomes negligible, and the wave behaves just like a plane wave. An acoustically large distance is typically defined as: $kr >> 1$ (The distance measured in wavelengths is large), beyond this distance, is what is commonly called **the far field** (assuming no reflections).

5.8 SPHERICAL WAVE INTENSITY AND POWER

The intensity of a spherical wave in the far field is the same as that for a plane wave:

$$I = \frac{1}{T}\int_0^T pu\,dt = \frac{\langle p^2 \rangle}{\rho c} = \frac{{p_{rms}}^2}{\rho c}$$

Power: A general expression for power, which is valid for any source

$$W = \int_S I\,dS$$

(note that power does not depend on distance from source).

If source is a non-directional, spherical radiator, the intensity is uniform over a sphere surrounding the source (surface area is $4\pi r^2$). The total radiated power is:

$$W = 4\pi r^2 I$$

Spherical Wave Summary:

In the far field (kr>>1):

Pressure p and Velocity u are in phase

Pressure: $p \propto 1/r$

Intensity: $I \propto 1/r^2 \propto p^2$

Impedance: $Z=\rho c$ (real)

In near field:

Complex Impedance

Complicated relationship between p, u and r

5.9 SOURCE DIRECTIONALITY

Most real sources do not radiate uniformly in all directions and are quite directional. We will quantify the directionality characteristics by the Directional Index, covered in the next section. The radiation from a cello is shown below. Areas of local energy recirculation are apparent very close to the cello surface (in near field). Farther away from the body, the intensity vectors start to behave more as you would expect from a point source.

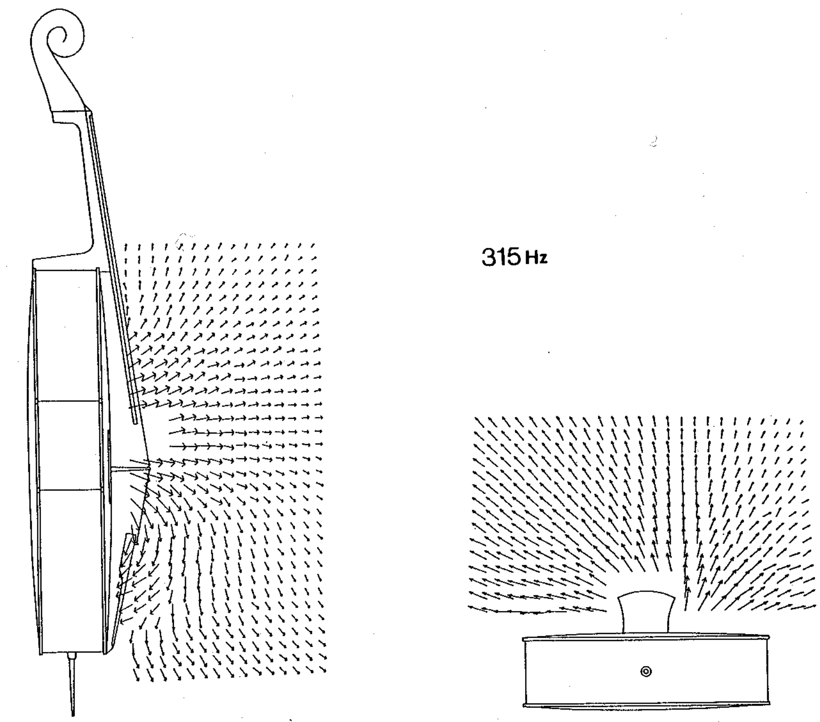


Figure 4. Radiation pattern from a cello, as measured by the two microphone technique. The magnitude and direction of the sound intensity are indicated by the length and direction of the arrows.

The degree of directionality depends on the size of the radiating surface relative to the acoustic wavelength. Sources which are physically large relative to the sound wavelength (ka >> 1) tend to be highly directional radiators. As seen in Figure 5, a 12" speaker for example, will be almost omnidirectional at 360 Hz (ka=1), but highly directional (indicated by the lobed pattern) at 3600 Hz (ka=10). This general trend is observed for most sources i.e. that ***low frequencies tend to radiate very uniformly, while high frequency radiation becomes highly directional***.

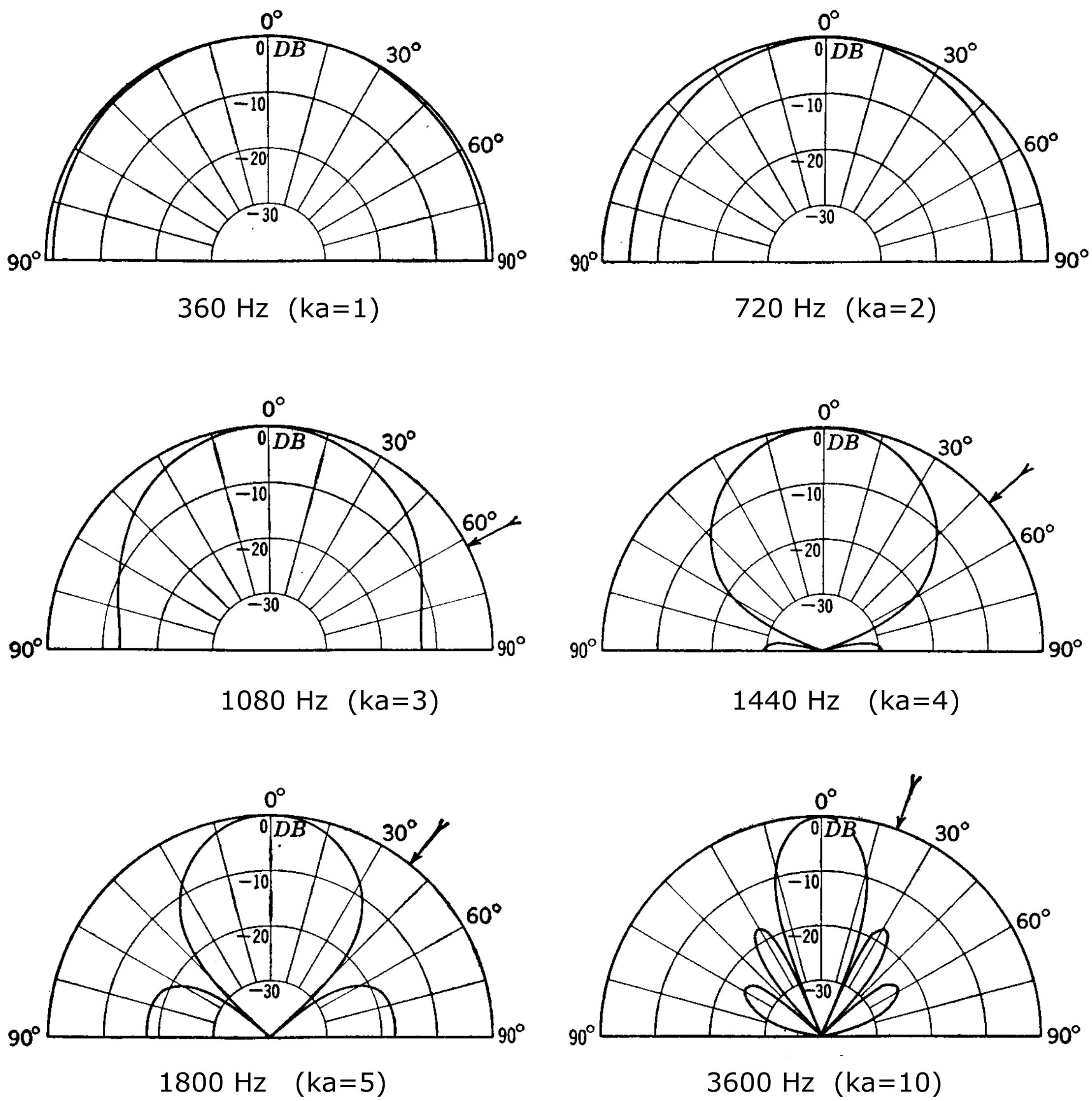


Figure 5. Directional radiation pattern from a 12″ circular piston in an infinite baffle – a good model for a loudspeaker. (a=radius of piston)

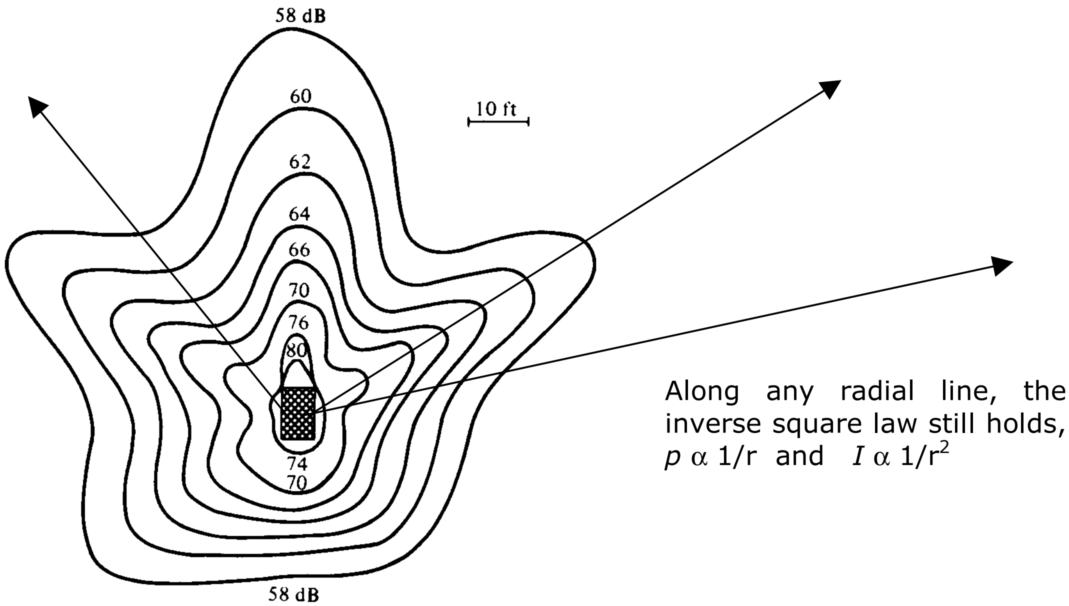


Figure 6. Sound pressure level contours around an electrical transformer

The radiation pattern from an electrical transformer is shown in Figure 6.

Even though these sources do not radiate uniformly, along any radial line outward from the source, in the far field, with no reflections, the sound pressure is still proportional to 1/r and the intensity is proportional to $1/r^2$. The constant of proportionality however, is different for each radial line.